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ABSTRACT: Using Leaver's continue fraction and time domain method, we study the wave dynamics of phantom scalar perturbation in a Schwarzschild black string spacetime. We find that the quasinormal modes contain the imprint from the wavenumber k of the fifth dimension. The late-time behaviors are dominated by the difference between the wavenumber k and the mass μ of the phantom scalar perturbation. For $k < \mu$, the phantom scalar perturbation in the late-time evolution grows with an exponential rate as in the four-dimensional Schwarzschild black hole spacetime. While, for $k = \mu$, the late-time behavior has the same form as that of the massless scalar field perturbation in the background of a black hole. Furthermore, for $k > \mu$, the late-time evolution of phantom scalar perturbation is dominated by a decaying tail with an oscillation which is consistent with that of the usual massive scalar field. Thus, the Schwarzschild black string is unstable only against the phantom scalar perturbations which satisfy the wavelength $\lambda > 2\pi/\mu$. These information can help us know more about the wave dynamics of phantom scalar perturbation and the properties of black string.

KEYWORDS: Classical Theories of Gravity, Black Holes

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Contents

1 Introduction

The current observations confirm that the expansion of the present Universe is speeding up rather than slowing down. This may indicate that our Universe contains dark energy which is an exotic energy component with negative pressure and constitutes about 72% of present total cosmic energy. The leading interpretation of such a dark energy is a cosmological constant with equation of state $\omega_x = -1$ [1]. Although the cosmological constant model is consistent with observational data, at the fundamental level it fails to be convincing. The vacuum energy density falls far below the value predicted by any sensible quantum field theory, and it unavoidably yields the coincidence problem, namely, "why are the vacuum and matter energy densities of precisely the same order today?". Therefore the dynamical scalar fields, such as quintessence [2], k-essence [3] and phantom field [4], have been put forth as an alternative of dark energy.

Comparing with other dynamical scalar fields, the phantom field model is more interesting because that it has a negative kinetic energy which can be appeared in the quantum particle creation processes in curved backgrounds [5] or can be motivated from S-brane constructs in string theory [6]. The presence of such a negative kinetic energy leads to that the null energy condition is violated and the equation of state ω_x of the phantom energy is less than -1. In the Einstein cosmology, the dynamical evolutions of phantom field [7–11] tell us that its energy density increases with the time and approaches to infinity in a finite time. In other words, the Universe dominated by phantom energy will blow up incessantly and arrive at a big rip finally, which is a future singularity with a strong exclusive force so that anything in the Universe including the large galaxies will be torn up. The thermodynamical properties of a phantom Universe are strange and such the universe has a negative entropy diverging near the big rip. Although the phantom energy possesses such exotic properties, it is favored by recent precise observational data involving CMB, Hubble Space Telescope and type Ia Supernova [13].

In the background of black hole spactimes, phantom field also exhibits some peculiar properties. E. Babichev and his co-workers [14] found that all black holes in the phantom

universe lose their masses to vanish exactly in the big rip. Therefore, it is possible that as a charged black hole absorbs the phantom energy the charge of a black hole will be larger than its mass. This is a serious threat to the cosmic censorship conjecture [15, 16] . In our previous paper [17], we studied that the wave dynamics of the phantom scalar perturbation in the Schwarzschild black hole spacetime. Our result shows that in the late-time evolution the phantom scalar perturbation grows with an exponential rate rather than decays as the usual scalar perturbations [18–22]. Moreover we also find that the phantom scalar emission will enhance the Hawking radiation of a black hole [23]. These results could help us to get a deeper understand about dark energy and black hole physics.

The discussions above tell us that comparing with other fields the phantom field plays the entirely different roles both in the cosmology and black hole physics. The recent investigations else show that the peculiar properties of phantom energy make it possible to be a candidate for exotic matter to construct wormholes [24]. However, all of the above investigations of phantom fields do not consider the case of black string. Therefore, what effects of the phantom field on black strings is still open. It is well known that black string is an important kind of black objects in the high dimensional gravity theories. In general, such a Schwarzschild black string suffers from the so-called Gregory-Laflamme instability, namely, long wavelength gravitational instability of the scalar type of the metric perturbations [25, 26]. The Gregory-Laflamme instability has been extensively studied in the last decade [27, 28] and the threshold values of the wavenumber k at which the instability appears are obtained [29]. For the usual scalar, electromagnetic and Dirac field perturbations, the Schwarzschild black string does not meet such a Gregory-Laflamme instability problem. Since the phantom field presents many peculiar behaviors in many fields, it is of interesting to study its dynamical evolution in the black string spacetime and to probe whether it presents some new behaviors. The main purpose of this paper is to investigate the quasinormal modes and late-time behavior of the phantom scalar perturbation in the Schwarzschild black string background through the well-known continue fraction [30] and time domain methods [31], and to discuss further the problems of instability of black string and a phantom scalar field.

The paper is organized as follows: in the following section we give the wave equation of phantom scalar field in the Schwarzschild black string spacetime. In sectionIII, we calculate the fundamental quasinormal modes of the phantom scalar perturbation by the continue fraction technique and plot the late-time evolution by time domain method. Finally in the last section we include our conclusions.

2   The wave equation of phantom scalar field in the Schwarzschild black string spacetime

Let us now to consider a Schwarzschild black string spacetime with the compact fifth z-coordinate, whose metric in the standard coordinate can be described by

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) - dz^2. \qquad (2.1)$$

The horizon event horizon is situated at $r = 2M$ and we assume that the $z-$direction is periodically identified by the relation $z = z + 2\pi R$.

In the background spacetime (2.1), the action of the phantom scalar perturbation with the negative kinetic energy term can be expressed as [4, 32]

$$S = \int d^4x dz \sqrt{g} \left[-\frac{R}{16\pi G} - \frac{1}{2}\partial_\mu \psi \partial^\mu \psi + V(\psi) \right]. \tag{2.2}$$

Here we take metric signature $(+ - - - -)$. The usual "Mexican hat" symmetry breaking potential has the form

$$V(\psi) = -\frac{1}{2}\mu^2 \psi^2 + \frac{\kappa}{4}\psi^4, \tag{2.3}$$

where μ is the mass of the scalar field and κ is the coupling constant. Here we treat the phantom scalar field as an external perturbation and suppose it does not change the metric of the background. As in refs. [17, 23], we only consider the case $\kappa = 0$ for conveniently. Thus, varying the action with respect to ψ, we obtain the wave equation for phantom scalar field in the curve spacetime

$$\frac{1}{\sqrt{g}}\partial_\mu(\sqrt{g}g^{\mu\nu}\partial_\nu)\psi - \mu^2\psi = 0. \tag{2.4}$$

Substituting eq. (2.1) into eq. (2.4), and separating variable

$$\psi = e^{-i\omega t}e^{ikz}\frac{R(r)Y_{lm}(\theta,\phi)}{r}, \qquad k = \frac{N}{R}, \qquad N \in Z, \tag{2.5}$$

we can obtain the radial equation for the scalar perturbation in the Schwarzschild black string spacetime

$$\frac{d^2 P(r)}{dr_*^2} + [\omega^2 - V(r)]R(r) = 0, \tag{2.6}$$

where r_* is the tortoise coordinate (which is defined by $dr_* = \frac{r}{r-2M}dr$) and the effective potential $V(r)$ reads

$$V(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2M}{r^3} + k^2 - \mu^2\right). \tag{2.7}$$

The form of the effective potential $V(r)$ (2.7) for the phantom scalar perturbation is similar to that in the Schwarzschild black hole spacetime [17], but there is an additional "mass" term k^2 in $V(r)$, which originates from the compact fifth dimension of a Schwarzschild black string. The presence of the term k^2 changes the behavior of the effective potential $V(r)$ of the phantom scalar field. With the increase of k, the peak height of the potential barrier increases. Moreover, at the spatial infinity the effective potential $V(r)$ approaches to a constant $k^2 - \mu^2$ rather than $-\mu^2$ in Schwarzschild black string spacetime. For the case k^2 is smaller than μ^2, we find that $V(r)$ is negative as r tends to infinity. It is similar to that in Schwarzschild black hole spacetime in which the negative limit $V(r)|_{r\to\infty}$ yields
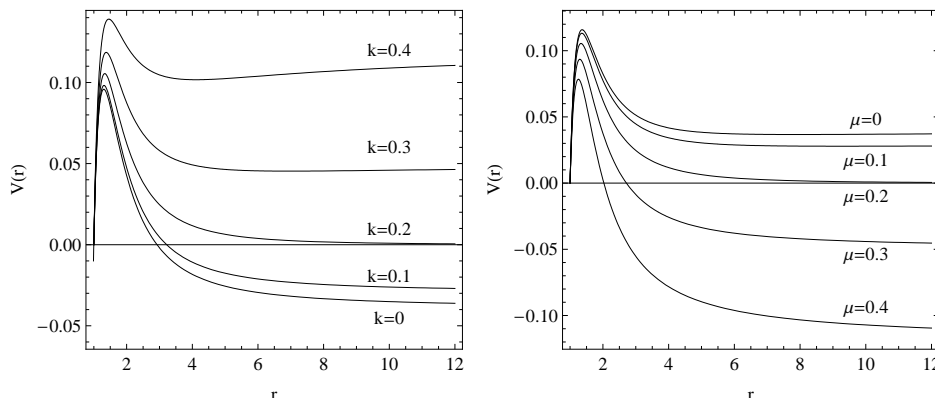
Figure 1. Variety of the effective potential $V(r)$ of phantom scalar perturbation with r. The left for fixed $\mu = 0.2$ and the right for fixed $k = 0.2$ in the Schwarzschild black string spacetime. Here $l = 0$ and $M = 0.5$.

that the phantom scalar perturbation in the late-time evolution grows with an exponential rate. However, for k^2 is larger than or equal to μ^2, the effective potential $V(r)$ (2.7) is no more negative again in the physical regime. In general, the positive potential means that the background spacetime is stable against an external perturbation. This implies also that the evolution of phantom scalar perturbation possesses some new properties in the Schwarzschild black string spacetime.

3 Evolution of phantom scalar perturbation in the Schwarzschild string spacetime

In this section we will study the wave dynamics of phantom scalar perturbation in the Schwarzschild string spacetime by using Leaver's continue fraction [30] and time domain method [31].

Let us now to calculate the fundamental quasinormal modes which are dominated in the late time oscillations. The boundary conditions on the wave function $R(r)$ of phantom scalar perturbation in the Schwarzschild black string spacetime can be expressed as

$$R(r) = \begin{cases} (r-1)^{-i\omega}, & r \to 1, \\[2mm] r^{\frac{i(\chi^2 + \omega^2)}{2\chi}} e^{i\chi r}, & r \to \infty, \end{cases} \tag{3.1}$$

where we set $2M = 1$ and $\chi = \sqrt{\omega^2 - k^2 + \mu^2}$. A solution to eq. (2.6) that has the desired behavior at the boundary can be written as

$$R(r) = r^{i(\chi+\omega) - \frac{i\mu^2}{2\chi}}(r-1)^{-i\omega}e^{i\chi r}\sum_{n=0}^{\infty} a_n\left(\frac{r-1}{r}\right)^n. \tag{3.2}$$

– 4 –

k	$\mu = 0$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$
0	0.22091-0.20979i	0.22094-0.20998i	0.22101-0.21056i	0.22114-0.21151i
0.01	0.22088-0.20960i	0.22091-0.20979i	0.22099-0.21037i	0.22111-0.21132i
0.02	0.22081-0.20902i	0.22083-0.20921i	0.22091-0.20979i	0.22104-0.21075i
0.03	0.22068-0.20805i	0.22071-0.20825i	0.22078-0.20883i	0.22091-0.20979i
0.04	0.22050-0.20668i	0.22053-0.20687i	0.22060-0.20746i	0.22073-0.20844i

Table 1. The fundamental ($n = 0$) quasinormal frequencies of phantom scalar field (for fixed $l = 0$) in the Schwarzschild black string spacetime.

Substituting (3.2) into (2.6), we find that the coefficients a_n of the expansion satisfy a three-term recurrence relation staring from $a_0 = 1$

$$\alpha_0 a_1 + \beta_0 a_0 = 0,$$
$$\alpha_n a_{n+1} + \beta_n a_n + \gamma_n a_{n-1} = 0, \qquad n = 1, \ 2, \ \ldots. \tag{3.3}$$

In terms of n and the black hole parameters, one find that the recurrence coefficients α_n, β_n and γ_n have the forms

$$\begin{aligned}
\alpha_n &= n^2 + (C_0 + 1)n + C_0, \\
\beta_n &= -2n^2 + (C_1 + 2)n + C_3, \\
\gamma_n &= n^2 + (C_2 - 3)n + C_4 - C_2 + 2,
\end{aligned} \tag{3.4}$$

where the intermediate constants C_n are the functions of the variables ω and χ

$$\begin{aligned}
C_0 &= 1 - 2i\omega, \\
C_1 &= -4 + i(4\omega + 3\chi) + \frac{i\omega^2}{\chi}, \\
C_2 &= 3 - i(2\omega + \chi) - \frac{i\omega^2}{\chi}, \\
C_3 &= \left(\frac{\omega + \chi}{\chi}\right)\left[(\omega + \chi)^2 + \frac{i(\omega + 3\chi)}{2}\right] - l(l+1) - 1, \\
C_4 &= -\left[\frac{(\omega + \chi)^2}{2\chi} + i\right]^2.
\end{aligned} \tag{3.5}$$

The boundary conditions are satisfied when the continued fraction condition on the recursion coefficients holds. The series in (3.3) converge for the given l. The frequency ω is a root of the continued fraction equation

$$\beta_n - \frac{\alpha_{n-1}\gamma_n}{\beta_{n-1} - \frac{\alpha_{n-2}\gamma_{n-1}}{\beta_{n-2} - \alpha_{n-3}\gamma_{n-2}/\ldots}} = \frac{\alpha_n \gamma_{n+1}}{\beta_{n+1} - \frac{\alpha_{n+1}\gamma_{n+2}}{\beta_{n+2} - \alpha_{n+2}\gamma_{n+3}/\ldots}}. \tag{3.6}$$

Numerical solutions of this algebraic equation give us the quasinormal spectrum of phantom scalar perturbations in the Schwarzschild black string spacetime.

k	$\mu = 0$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$
0	0.58587-0.19532i	0.58581-0.19537i	0.58563-0.19553i	0.58534-0.19579i
0.01	0.58593-0.19527i	0.58587-0.19532i	0.58569-0.19548i	0.58540-0.19574i
0.02	0.58611-0.19511i	0.58605-0.19516i	0.58587-0.19532i	0.58558-0.19558i
0.03	0.58641-0.19485i	0.58635-0.19490i	0.58617-0.1950i	0.58587-0.19532i

Table 2. The fundamental ($n = 0$) quasinormal frequencies of phantom scalar field (for fixed $l = 1$) in the Schwarzschild black string spacetime.

k	$\mu = 0$	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$
0	0.96729-0.19352i	0.96724-0.19354i	0.96711-0.19360i	0.96690-0.19370i
0.01	0.96733-0.19350i	0.96729-0.19352i	0.96716-0.19358i	0.96694-0.19368i
0.02	0.96746-0.19344i	0.96742-0.19346i	0.96729-0.19352i	0.96707-0.19362i
0.03	0.96768-0.19333i	0.96763-0.19335i	0.96750-0.19341i	0.96729-0.19352i

Table 3. The fundamental ($n = 0$) quasinormal frequencies of phantom scalar field (for fixed $l = 2$) in the Schwarzschild black string spacetime.
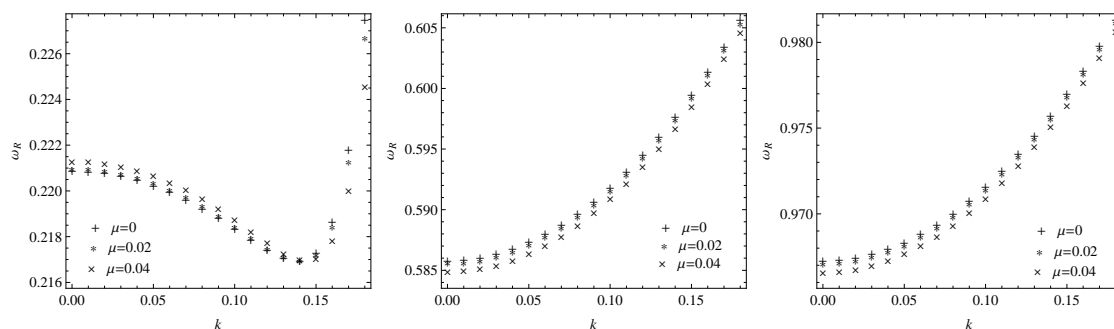

Figure 2. Variety of the real part quasinormal frequencies ($n = 0$) for the phantom scalar perturbations with k for different μ. The left, middle and right are for $l = 0$, $l = 1$ and $l = 2$.

In tables 1–3, we list the fundamental quasinormal frequencies of phantom scalar perturbation field for fixed $l = 0 \sim 2$ in the Schwarzschild black string spacetime. From the tables 1–3 and figures 2 3, one can find that for $l = 0$ with the increase of k the real part first decreases and then increases, but with the increase of μ it is in inverse. Moreover, for the $l = 1$ and $l = 2$ the real part increases with k and decreases with μ. Therefore the effect of k on the quasinormal frequencies is different from that of the mass μ of phantom scalar perturbation. The main reason is that the sign of k^2 in the $V(r)$ is opposite to that of μ^2, which yields that their effects on the potential are different. The absolute value of imaginary parts for all l decrease with k and increase with μ. This can be explained by that the smaller k and larger μ leads to the lower peak of the potential and thus it is easier for the wave to be absorbed into the black hole.

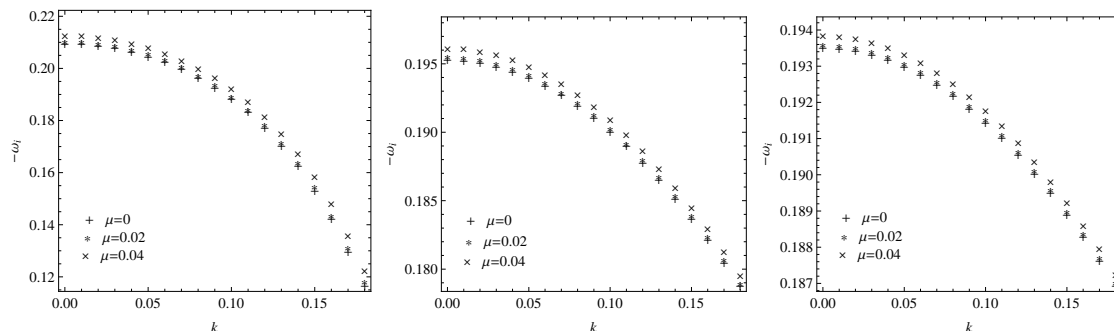We are now in a position to study the late-time behavior of phantom scalar perturba-

Figure 3. Variety of the imaginary part quasinormal frequencies ($n = 0$) for the phantom scalar perturbations with k for different μ. The left, middle and right are for $l = 0$, $l = 1$ and $l = 2$.

tions in the Schwarzschild black string spacetime by using of the time domain method [31]. Using the light-cone variables $u = t - r_*$ and $v = t + r_*$, the wave equation

$$-\frac{\partial^2 \psi}{\partial t^2} + \frac{\partial^2 \psi}{\partial r_*^2} = V(r)\psi, \tag{3.7}$$

can be rewritten as

$$4\frac{\partial^2 \psi}{\partial u \partial v} + V(r)\psi = 0. \tag{3.8}$$

The two-dimensional wave equation (3.8) can be integrated numerically by using the finite difference method suggested in [31]. In terms of Taylor's theorem, it is discretized as

$$\psi_N = \psi_E + \psi_W - \psi_S - \delta u \delta v V \left(\frac{v_N + v_W - u_N - u_E}{4}\right)\frac{\psi_W + \psi_E}{8} + O(\epsilon^4) = 0, \quad (3.9)$$

where we have used the following definitions for the points: N: $(u + \delta u, v + \delta v)$, W: $(u + \delta u, v)$, E: $(u, v + \delta v)$ and S: (u, v). The parameter ϵ is an overall grid scalar factor, so that $\delta u \sim \delta v \sim \epsilon$. Considering that the behavior of the wave function is not sensitive to the choice of initial data, we set $\psi(u, v = v_0) = 0$ and use a Gaussian pulse as an initial perturbation, centered on v_c and with width σ on $u = u_0$ as

$$\psi(u = u_0, v) = e^{-\frac{(v - v_c)^2}{2\sigma^2}}. \tag{3.10}$$

Here we examine numerically the late-time behaviors of phantom scalar perturbations in the Schwarzschild black string spacetime. In figures (4) and (5), we plot the evolutions of the phantom scalar perturbations for fixed $l = 0$ and $l = 1$ respectively. For all l, the late-time behaviors of phantom scalar perturbation depend heavily on the sign of the quantity $k^2 - \mu^2$. When it is negative (i.e. $k^2 < \mu^2$), we find the phantom scalar field after undergoing the quasinormal modes grow with exponential rate. As in the Schwarzschild black hole spacetime, the asymptotic behaviors of the wave function have the form

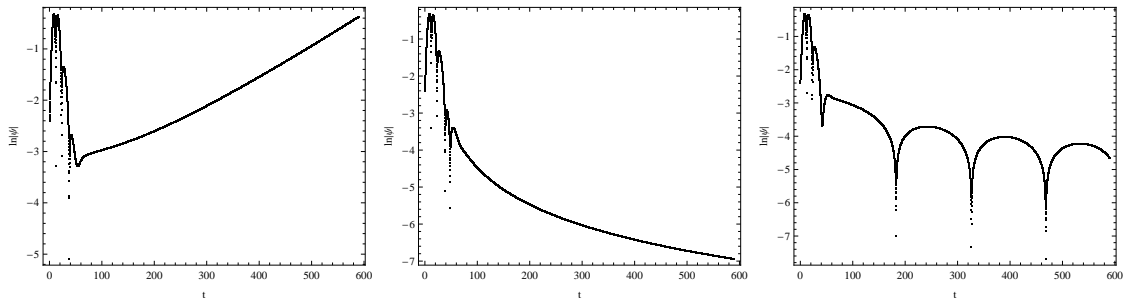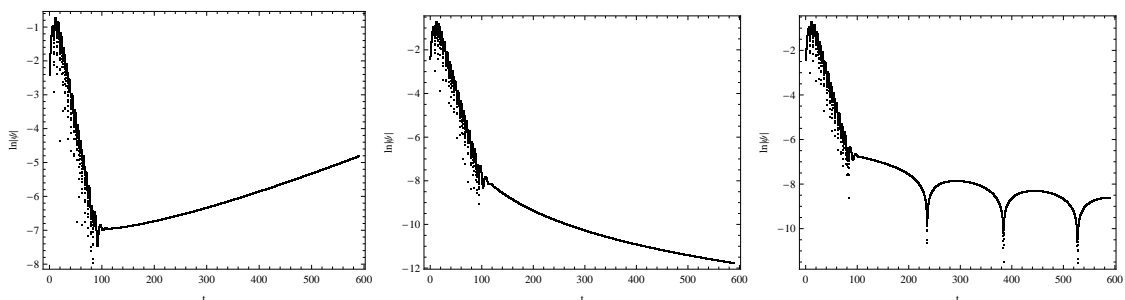$$\psi \sim e^{\alpha\sqrt{\mu^2 - k^2}t - 4l - \beta}, \tag{3.11}$$

Figure 4. The late-time behaviors of the phantom scalar perturbations for fixed $l = 0$ and $\mu = 0.02$, the left, middle and right are for $k = 0.01$, 0.02 and 0.03, respectively. The constants in the Gauss pulse (3.10) $v_c = 10$ and $\sigma = 3$.

Figure 5. The late-time behaviors of the phantom scalar perturbations for fixed $l = 1$ and $\mu = 0.02$, the left, middle and right are for $k = 0.01$, 0.02 and 0.03, respectively. The constants in the Gauss pulse (3.10) $v_c = 10$ and $\sigma = 3$.

where α and β are two numerical constant. This behavior can be attributed to that the effective potential is negative at the spatial infinity and the wave outside the black hole gains energy from the spacetime [33]. Moreover, the exponential growth of the wave function means also that in this case the Schwarzschild black string spacetime is unstable against the external phantom scalar perturbation. When the wavenumber k is equal to the mass μ of the phantom scalar perturbation, we find that it decays without any oscillation, which is similar to that of usual massless scalar field in the Schwarzschild black hole spacetimes [31, 34–36]. It is not surprising because that in this case the effective potential (2.7) has the same form as that of the usual massless scalar field. When the quantity $k^2 - \mu^2$ is positive, the figures (4) and (5) tell us the behaviors of the phantom scalar perturbation have the form $t^{-\gamma} \sin(\sqrt{k^2 - \mu^2}\, t)$. This implies that the phantom scalar field decays with the oscillatory inverse power-law behavior which is similar to that of the usual massive scalar perturbations in a black hole spacetime [21, 22]. These properties of phantom scalar field have not been observed elsewhere.

From the discussion above, we know that in the case $k^2 \geq \mu^2$ the Schwarzschild black string spacetime is stable when it suffers from the external phantom-like perturbations. Thus there exists a threshold values of the wavenumber k at which the stability of phantom perturbations appears. In other words, only the phantom scalar perturbations whose wavelengths satisfy $\lambda > 2\pi/\mu$ is unstable in Schwarzschild black string spacetime. For the

smaller μ, the range of the wavelength λ of the phantom field is stable becomes wider. For the fixed μ, only the phantom perturbation has the longer wavelength is unstable. This result is consistent with that of the Gergory-Laflamme instability which is obtained in the evolution of the scalar type of the gravitational perturbation in this background. For the usual scalar perturbations, the quantity $k^2 - \mu^2$ in the effective potential $V(r)$ (2.7) is replaced by $k^2 + \mu^2$. Thus for the usual scalar perturbations there does not exist such threshold value for the wavenumber k and the perturbations are always stable in the Schwarzschild black string spacetime.

4 Summary

In this paper we examined the dynamical evolution of the phantom scalar perturbation in the Schwarzschild black string spacetime. We find that the quasinormal modes and late-time behaviors contain the imprint from the fifth dimension. The form of the late-time evolution is determined by the difference between the wavenumber k and the mass μ of the phantom scalar perturbation. For $k < \mu$, the phantom scalar perturbation in the late-time evolution grows with an exponential rate as in the four-dimensional Schwarzschild black hole spacetime. While, for $k = \mu$, the late-time behavior has the same form as that of the massless scalar field perturbation in the background of a black hole. Furthermore, for $k > \mu$, the late-time evolution of phantom scalar perturbation is dominated by a decaying tail with an oscillation which is consistent with that of the usual massive scalar field. These information can help us know more about the wave dynamics of phantom scalar perturbation and the properties of black string. It would be of interest to generalize our study to other spacetimes. Work in this direction will be reported in the future.

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